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A problem on maximum dense subgraphs

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Let $G = (V, E)$ be a simple connected graph. For $m = 1, 2, \ldots, |V|$ denote by $E(m)$ the maximum number of edges in an induced subgraph of G of order m. We call this problem Edge-Isoperimetric Problem (abbr. EIP), and the vertex sets with maximum number of induced edges are called optimal sets. Further denote $\delta(m) = E(m) - E(m-1)$ for $m = 1, 2, \ldots, |V|$, and for $k = \max_m \delta(m)$ define $\delta'(m) = k - \delta(m)$ for $m = 1, 2, \ldots, |V|$.

We say that the δ-sequence $\delta(1), \ldots, \delta(m)$ is symmetric if $\delta(m) = \delta'(|V| - m + 1)$ for $m = 1, \ldots, |V|$. It is easy to show that if G is regular, then its δ-sequence is symmetric.

Conjecture 1: If the δ-sequence of G is symmetric, then G is regular.

All known to me results related to solving EIP for cartesian powers of a given graph deal with powers of regular graphs, see, e.g., [1],[2]. It would be equally interesting either to prove this conjecture or construct a counterexample. Currently, I believe in its validity.

If a counterexample is found, then what I am really interested in is to prove/disprove Conjecture 1 for graphs satisfying the following additional condition.

We say that G admits nested solutions (NS) in EIP is there exists a total order on the vertex set V such that for every $m = 1, 2, \ldots, |V|$ the set of the first m vertices of G in this order is an optimal set.

Conjecture 2: If G admits NS in EIP and its δ-sequence is symmetric, then G is regular.

References

[1] S.L. Bezrukov and R. Elsässer. Edge isoperimetric problems for cartesian powers of regular graphs. *Theor. Comput. Sci.*, 307:473–492, 2003.

[2] S.L. Bezrukov Edge isoperimetric problems on graphs. in: Graph Theory and Combinatorial Biology, Bolyai Soc. Math. Stud. 7, L. Lovasz, A. Gyarfas, G.O.H. Katona, A. Recski, L. Szekely eds., Budapest 1999, 157-197.