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# Vertex Cover with Neighborly Help under Edge Deletion

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In the NP-hard optimization problem Vertex Cover, we are asked to find a minimum vertex cover in a given graph. (The given graph is simple; that is, it is undirected, unweighted, and has neither loops nor parallel edges). We consider Vertex Cover in the recently introduced *Neighborly Help* model [1], which allows us to query an oracle for optimal solutions to *neighbor instances*. The notion of a neighbor needs to be specified for every problem. We examine the local modification of an edge deletion: We consider as neighbors exactly those graphs that result by removing from the given graph a single edge.

Given a graph, we may therefore query the oracle for a minimum vertex cover to any one-edge-deleted subgraph of our choice.

**One Query: NP-Hard.** Restricted to a single query, Vertex Cover remains NP-hard.

**Two and More Queries: Open.** It remains open, however, whether the becomes polynomial-time solvable when two queries or even arbitrarily many queries are allowed.

Note that Vertex Cover under edge deletion is equivalent to Clique under edge addition. Note that it might be possible for a from NP-hardness to P to occur between $k$ and $k+1$ for some $k > 1$.

## References

[1] Elisabet Burjons, Fabian Frei, Edith Hemaspaandra, Dennis Komm, and David Wehner. Finding Optimal Solutions With Neighborly Help. In *Proc. MFCS*, volume 1382 of *LIPIcs*, pages 78:1–78:14, 2019. The full version including the appendices is available at https://arxiv.org/abs/1906.10078.

Table 1: This table is taken from the paper that introduced the Neighborly Help model [1]; all referenced theorems can be found in the full version of this paper on arXiv. The table provides an overview of the known results regarding the hardness of Colorability and Vertex Cover in the Neighborly Help model for the most common definitions of a local modification. The $v$ stands for a vertex and the $e$ stands for an edge. The question mark indicates an interesting open problem. The results in the vertex-addition columns are trivial; see Theorem 16. The NP-hardness results for the 1-query case all follow from rather simple Turing reductions; see Theorem 17.

| No. of Queries | Colorability | | | | Vertex Cover | | | |
|---|---|---|---|---|---|---|---|---|
| | Add $v$ | Delete $v$ | Add $e$ | Delete $e$ | Add $v$ | Delete $v$ | Add $e$ | Delete $e$ |
| 1 | P | NP-hard | NP-hard | NP-hard | P | NP-hard | NP-hard | NP-hard |
| 2 or more | P [Thm. 16] | NP-hard [Thm. 4] | P [Thm. 10] | NP-hard [Thm. 6] | P [Thm. 16] | P [Thm. 12] | P [Thm. 19] | ? |