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Readability of bipartite graphs

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1 The problem, briefly

Given two finite sets of finite strings S, T, the *bipartite overlap graph* of (S,T) is the bipartite graph $B(S,T)$ with parts $\{u_s : s \in S\}$ and $\{v_t : t \in T\}$ such that u_s is adjacent to v_t if and only if strings s and t overlap (that is, some nonempty suffix of s equals some prefix of t). It is known that every bipartite graph is isomorphic to some bipartite overlap graph. Therefore, we can define the *readability* of a bipartite graph G as

$$r(G) = \min\left\{ \max_{s \in S \cup T} |s| : G \cong B(S,T) \right\},$$

where $\cong$ denotes the graph isomorphism relation.

Open problem ([2]): Determine the computational complexity of the problem of computing the readability of a given bipartite graph G.

2 The problem, in some more detail

Let $G = (V,E)$ be a bipartite graph with a given bipartition of its vertex set $V(G) = V_s \cup V_p$. (We will also use the notation $G = (V_s, V_p, E)$.) A *labeling* of G is a function ℓ assigning a string to each vertex such that all strings have the same length, denoted by $len(\ell)$. Given two strings x and y, we say that x *overlaps* y if there is a nonempty suffix of x that is equal to a nonempty prefix of y. An *overlap labeling of* G is a labeling ℓ of G such that for all $u \in V_s$ and $v \in V_p$, $(u,v) \in E$ if and only if the strings $\ell(u)$ and $\ell(v)$ overlap (that is, if some nonempty suffix of $\ell(u)$ equals some nonempty prefix of $\ell(v)$).

The *readability* of G, denoted by $r(G)$, is the smallest nonnegative integer r such that there exists an overlap labeling of G of length r. It follows from results of Braga and Meidanis [1] that

every bipartite graph G has $r(G) \leq 2^{\Delta(G)} - 1$, where $\Delta(G)$ denotes the maximum degree of a vertex in G (see [2]). (In fact, every bipartite graph admits an overlap labeling over the binary alphabet.)

Problem 1 (Chikhi et al. [2]) *Determine the computational complexity status of computing the readability of a given bipartite graph $G = (V_s, V_p, E)$.*

Remarks: The cases $r(G) \leq 1$ and $r(G) \leq 2$ are known to be polynomial. For every constant $k \geq 3$, the complexity of the problem of testing if $r(G) \leq k$ is open. Note that there is no restriction on alphabet size. (However, problems analogous to Problem 1 seem to be open also for fixed-size alphabets.)

References

[1] M. D. V. Braga and J. Meidanis. An algorithm that builds a set of strings given its overlap graph. In *LATIN 2002: Theoretical Informatics, 5th Latin American Symposium, Cancun, Mexico, April 3-6, 2002, Proceedings*, pages 52–63, 2002.

[2] R. Chikhi, P. Medvedev, M. Milanič, and S. Raskhodnikova. On the readability of overlap digraphs. In *Combinatorial Pattern Matching - 26th Annual Symposium, CPM 2015, Ischia Island, Italy, June 29 - July 1, 2015, Proceedings*, pages 124–137, 2015.