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Straight-Line Orthogonal Drawings of Complete Ternary Trees in Near-Linear Area

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A *rooted tree* is a connected acyclic graph with a distinguished node called the *root*. A rooted tree is *binary* (*ternary*, respectively) if it has maximum degree 3 (4, respectively) and its root has maximum degre 2 (3, respectively). A tree is *complete* if every non-leaf node has the maximum number of children and every root-to-leaf path has the same length. Figure 1 shows a complete ternary tree of depth six.

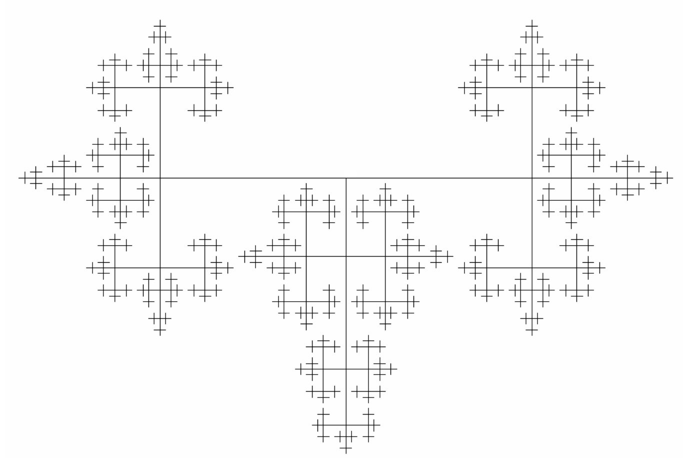

Figure 1: A minumum area drawing of a complete ternary tree of depth six satisfying the subtree separation property computed with the approach in [4].

A *planar straight-line orthogonal drawing* of a tree represents each vertex as a point in the plane and each edge either as a horizontal or as a vertical straight-line segment, so that no two edges cross. Since planar straight-line orthogonal drawings of binary and ternary trees always exist, most research efforts have been devoted to the construction of drawings with small area. This is usually formalized by requiring nodes to lie at *grid* points, i.e., points with integer coordinates, and by defining the *width* and *height* of a drawing as the number of grid columns and rows intersecting it, respectively, and the *area* as the width times the height.

A drawing algorithm draws in *near-linear* area if it produces drawings of n-node trees in area $O(n \cdot f(n))$, where $f(n)$ is a sub-polynomial function of n. For binary trees several drawing algorithms are known to guarantee near-linear area, such as the $O(n \log \log n)$ area of [3, 9], recently improved by Chan [2] to $n \cdot 2^{O(\log^* n)}$, where $\log^* n$ denotes the iterated logarithm. For complete binary trees $O(n)$ area is sufficient [5, 8].

Frati proved [6] that ternary trees and complete ternary trees admit planar straight-line orthogonal drawings in $O(n^{1.631})$ and $O(n^{1.262})$ area, respectively. The former bound was improved to $O(n^{1.576})$ by Covella et al. [4]; the latter bound was improved to $O(n^{1.118})$ by Ali [1]. The open problem we propose for the IWOCA 2018 Open Problem Session is that of devising a drawing algorithm that guarantees near-linear area for ternary trees or proving that such an area bound cannot be achieved. Both results would be interesting even if restricted to complete ternary trees.

Furter, a drawing of a tree is said to satisfy the *subtree separation property* if the smallest axis-parallel rectangles enclosing the drawings of any two node-disjoint subtrees do not overlap. This property has been frequently considered in the tree drawing literature [3, 7, 4]. In the paper [4], presented at IWOCA 2018, we conjecture that complete ternary trees do not admit drawings that satisfy the subtree separation property in near-linear area.

Conjecture 1 ([4]) *There exists a constant $\varepsilon > 0$ such that n-node complete ternary trees require $\Omega(n^{1+\epsilon})$ area in any planar straight-line orthogonal drawing satisfying the subtree separation property.*

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