Pancake flipping: The Big 3

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presented at IWOCA 2021
06 July 2021

Pancake flipping for permutations: A stack of n different sized pancakes can be modeled by a permutation. A *flip* of j pancakes corresponds to a prefix reversal of length j in the permutation. If the pancakes are burnt on one side, then they can be modeled by signed permutations and a flip also changes the signs of the reversed elements. The following problems correspond to finding Hamilton paths (or cycles) in their corresponding underlying graphs.

1. **Big 3**: Construct a flip-Gray code for permutations of order $n > 2$ where you are only allowed flips of size $n-2$, $n-1$, n.

2. **Big 3 (burnt)**: Construct a flip-Gray code for signed permutations (burnt pancakes) of order n where you are only allowed flips of size $n-2$, $n-1$, n.

3. Construct a flip-Gray code for permutations of order n where you are only allowed flips of size $\mathbf{2}$, $n-1$, n.

Since the corresponding graphs are connected, by showing no such Gray code exists would be contrary to a famous conjecture of Lovász stating that every connected Caley graph has a Hamilton cycle). Does the Big 3 problem generalize to coloured permutations?

References

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