International Workshop On Combinatorial Algorithms
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Problems on Minimal Pancyclic Graphs

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presented at IWOCA 2014

A *pancyclic* graph on v vertices is called *pancyclic* if it contains cycles of every length from 3 to v. An obvious question is, what is the minimum number of edges in a pancyclic graph with v vertices? Define $m(v)$ to be the minimum value. A pancyclic graph with v vertices and $m(v)$ edges is called minimal pancyclic.

Two obvious questions present themselves.

1. Is it always true that $m(v) \leq m(v+1)$?
   (It is hard to imagine otherwise, but a proof would be nice.)
2. Find a good upper bound for $m(v)$.

As complete graphs are pancyclic, there is an answer to Question 2. The paper [3] claims to give exact values of $m(v)$ for all v, but they have been proven wrong; for example, it is claimed that $m(13) = 17$, but an example with $m(13) = 16$ is given in [1]. (Other exact values, up to $v = 37$, are given in [1] and [2]). Some authors still cite the results in [3] as upper bounds on $m(v)$, but the proofs therein are not correct as written.

References

[1] J. C. George, A. Marr and W. D. Wallis, "Minimal pancyclic graphs," *J. Combin. Math. Combin. Comput.* **86** (2013), 125–133.

[2] S. Griffin, "Minimal pancyclicity," *to appear*.

[3] M. R. Sridharan, "On an extremal problem concerning pancyclic graphs", *J. Math. Phys. Sci.* **12** (1978), 297–306.